Regular Expressions

Lecture 10
Sections 3.1 - 3.2

Robb T. Koether

Hampden-Sydney College

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Outline

1. Regular Expressions
2. Equivalence of Regular Expressions and DFAs
3. Assignment

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Regular Expressions

- Regular expressions are like algebraic expressions except that they describe regular languages.
- Examples:
  - $\mathbf{a} \cdot \mathbf{a}^*$
  - $(\mathbf{a} \cdot (\mathbf{a} + \mathbf{b}))^*$
  - $(\mathbf{b} + \mathbf{a} \cdot \mathbf{b}^* \cdot \mathbf{a})^*$

Regular Expressions

Definition (Basic regular expressions)

The basic regular expressions are

- The symbols of the alphabet Σ
- λ
- $\varnothing$

Regular Operators

Definition (Regular operators)

Definition (Regular expression)

A regular expression is one of the basic regular expressions or

- $r_1 + r_2$
- $r_1 \cdot r_2$ (or $r_1 r_2$)
- r_1^*
- (r_1)

where r_1 and r_2 are regular expressions.

The regular operators are union, concatenation, and star.

- Regular expressions are defined recursively.

Language of a Regular Expression

Definition (Language of a regular expression)

The language of a regular expression is defined as follows.

- $L(a) = \{a\}$ for every $a \in \Sigma^*$.
- $L(\lambda) = \{\lambda\}$
- $L(\varnothing) = \varnothing$
- $L(r_1 + r_2) = L(r_1) \cup L(r_2)$
- $L(r_1 \cdot r_2) = L(r_1)L(r_2)$
- $L(r_1^*) = (L(r_1))^*$
- $L((r_1)) = L(r_1)$

where r_1 and r_2 are regular expressions.

Language of a Regular Expression

- Examples

Language of a Regular Expression

- Examples
  - $$\begin{aligned} L(\mathbf{a}+\mathbf{b}) &= L(\mathbf{a}) \cup L(\mathbf{b}) \\ &= \{\mathbf{a}\} \cup \{\mathbf{b}\} \\ &= \{\mathbf{ab}\}. \end{aligned}$$

Language of a Regular Expression

- Examples
  -
$$\begin{aligned} L(\mathbf{a}+\mathbf{b}) &= L(\mathbf{a}) \cup L(\mathbf{b}) \\ &= \{\mathbf{a}\} \cup \{\mathbf{b}\} \\ &= \{\mathbf{ab}\}. \end{aligned}$$
  -
$$\begin{aligned} L(\mathbf{a}\cdot\mathbf{b}^*) &= L(\mathbf{a})L(\mathbf{b}^*) \\ &= \{\mathbf{a}\}\{\mathbf{b}\}^* \\ &= \{\mathbf{a}\}\{\lambda, \mathbf{b}, \mathbf{bb}, \mathbf{bbb}, \ldots\} \\ &= \{\mathbf{a}, \mathbf{ab}, \mathbf{abb}, \mathbf{abbb}, \ldots\}. \end{aligned}$$

Examples

Example (Regular expressions)

Write regular expressions for the following regular languages.

- All strings ending with **a**.
- All strings containing **aba**.
- All strings containing **aba** or **bab**.
- All strings containing **aba** and **bab**.
- All strings not containing **aaa**.

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1 Regular Expressions

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Theorem

A language is regular if and only if it is the language of some regular expression.

Proof ($\Leftarrow$).

- The basic languages $L(a)$, $L(\lambda)$, and $L(\varnothing)$ are regular.
- Union, concatenation, and Kleene star of regular languages are regular.
- Therefore, the language of a regular expression is regular.

$\square$

Regular Languages and Regular Expressions

Proof ($\rightarrow$), beginning.

- Let L be a regular language.
- We need to construct a regular expression r such that $L(r) = L$.
- We will use a generalized transition graph (GTG).

$\square$

Definition

A generalized transition graph is like a regular transition graph except that the labels are regular expressions. To make a transition from one state to another, we must read a *string* that matches the regular expression labeling that transition.

Converting a DFA to a Regular Expression

Proof, continued.

- Begin with a transition diagram for L.
- Replace each label (symbol) with the equivalent regular expression.
- Add a new start state that has no transitions into it, but has one λ-move from it to the original start state.
- Add a new accept state that has no transitions out of it, but has λ-moves into it from all of the original accept states.
- Remove all states from which the accept state is inaccessible.

$\square$

Converting a DFA to a Regular Expression

Proof, continued.

- Note that the number of states in the GTG is at least 3.
- The proof will reduce the number of states down to 2, at which point we will have the regular expression.

□

Converting a DFA to a Regular Expression

Proof, continued.

- Choose a non-initial, non-final state q to be removed.
- For every state p with a transition into q and for every state r with a transition from q, create a transition from p to r, as follows.
- Let
  - r_1 be the label on the transition $p \to q$.
  - r_2 be the label on a loop $q \to q$, if there is one.
  - r_3 be the label on the transition from $q \to r$.
- Apply the label $r_1 r_2^* r_3$ to the new transition.

□

Converting a DFA to a Regular Expression

Proof, concluded.

- After doing this for every combination of transitions into q and out of q, remove state q and all of its transitions.
- Repeat this process until only the initial and final states remain.
- The label on that single remaining transition is the regular expression.

$\square$

Example

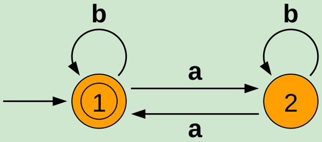
Example (Converting a DFA to a regular expression)

- Find a regular expression for the language

$$L = \{w \mid w \text{ has an even number of } \mathbf{a}\text{'s}\}.$$

Example

Example (Converting a DFA to a regular expression)



Example

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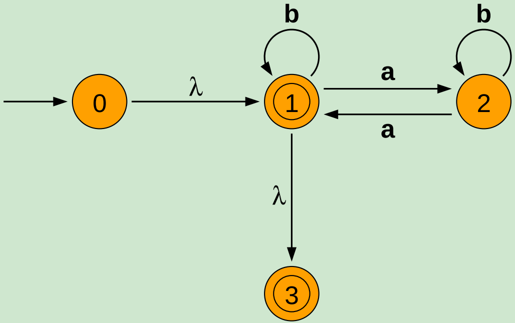

Example

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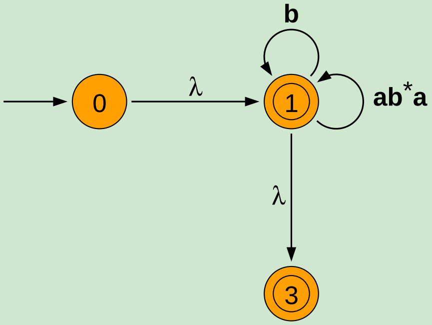

Example

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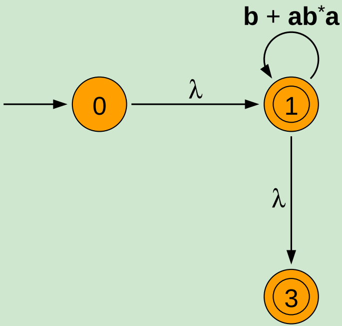

Example

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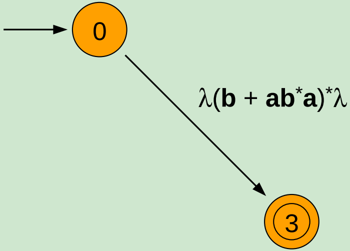

Example

Example (Converting a DFA to a regular expression)

- Therefore,

$$L((\mathbf{b} + \mathbf{ab}^*\mathbf{a})^*) = \{w \mid w \text{ has an even number of } \mathbf{a}\text{'s}\}.$$

- This regular expression "parses" the string

$$\mathbf{babbaababbbaab}$$

as

$$\mathbf{b|abba|aba|b|b|b|aa|b.}$$

Example

Example (Converting a DFA to a regular expression)

- Find regular expressions for the following languages
  - All strings containing an odd number of **a**'s.
  - All strings containing an even number of **a**'s and an even number of **b**'s.
  - All strings that do not contain **aaa**.

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Assignment

- p. 78: 3, 5, 11, 21, 22, 26, 27.
- p. 90: 1, 7, 10, 12b, 15ac.
- Write a regular expression that will match *****any text*****, where *any text* does not include the string ****.